Laboratory 2

Electronics Engineering 3210

Fourier Series Reconstruction

Purpose:

This lab gives students a clearer understanding of the Fourier series by having them reconstruct time-domain signals from Fourier coefficients. The lab also demonstrates the Gibbs Phenomenon, error signals and signal power.

Preliminary:

Part 1: Study Example 3.5 in the text. Assuming the amplitude, A, in that example is 1, then the Fourier series reconstruction of the triangle wave in that example is (note that $\omega_0=\pi$):

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi t) + b_n \sin(n\pi t)$$

where

$$a_n = 0 \text{ and } b_n = \frac{8}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

Of course, MATLAB cannot reconstruct a continuous time signal, but if we compute discrete points at a step size of about $1/(16f_{max})$, we will get a fairly good representation of the continuous signal. (f_{max} is the maximum frequency component of the signal.) In this case, we will be using a maximum of 25 harmonics (n<=25), so $f_{max} = 25\pi/2\pi$ or 12.5Hz. A good choice of step size, then, is 0.005. (Since the period of this signal is 2, you will want about 400 data points to display one period.)

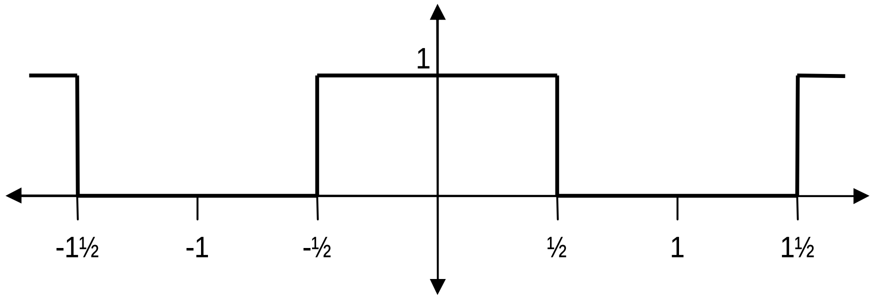
Write a MATLAB script to compute and plot one period of $f_m(t)$ by adding up to m = 25 sine and cosine waves:

$$f_m(t) = a_0 + \sum_{n=1}^{m} a_n \cos(n\pi t) + b_n \sin(n\pi t)$$

The script must also compute and plot the error signal $e(t) = f_m(t) - f(t)$. (You can use the same graph but plot in a different color.) Finally, the script should compute and output the power of the error signal:

$$P_e = \frac{1}{T}\sum e^2(t)\Delta t$$

Part 2: Find the Fourier coefficients for the square-wave signal depicted below. It may be useful to study Example 3.4 of the text.



Be prepared to modify your MATLAB script from Part 1 to plot the reconstructed signal, the error signal, and to compute the error power for this square-wave.

Procedure:

Write a title and short description of this lab on a new page of your lab book. Make an entry in the table of contents for this lab.

Run your script and plot the results for m = 5, 11 and 25. Print the results in each case and affix them to your lab book. Be sure to record the error power. What can you say about the quality of the reconstructed signal, $f_m(t)$ and the power of the error signal as m increases?

Modify your script to use the square-wave shown above and repeat the procedure, but also describe in your lab book the behavior of the reconstructed signal, $f_m(t)$ at the points of discontinuity (this behavior will become clear if you run your script with m = 50 and m = 100). Describe how this behavior (the Gibbs Phenomenon) changes as m increases. (i.e. does it change in amplitude, duration, frequency, or in some other way?)

Write a conclusion in your lab book that summarizes what you have observed.